Parallel Algorithms

1.0 Question One

Show how to compute the expression in parallel using “tree contraction”

$((5 + 4) \times 2) \times (((4 - 2) - (2 + 1) \times 2) \times 2 + 1)$

Show all intermediate steps how tree contraction is done.

How many operations are required to evaluate the expression?

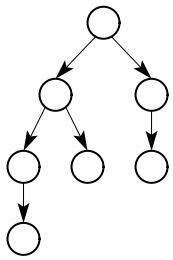
2.0 Question 2

Given a Hypercube of 2^k elements, suppose that a list $A[0], \ldots, A[i]$ is stored in $PE[0], \ldots, PE[i]$ where $i < 2^k$

Describe a $O(k)$ time algorithm which reverses the list, That is, the results should be that $PE[0]$ has $A[i]$, $PE[1]$ has $A[i-1]$ etc.

3.0 Question3

Demonstrate how to compute the number of decendent of a binary tree. Demonstrate your algorithm with this binary tree.



4.0 Question 4

Draw a de Bruijn's graph for 16 nodes

Can you find a hamaltonian path for this graph?

5.0 Question 5

A) Show how to solve the following recurrence equation using parallel prefix.
$z_i = sqrt({z_{i-1}}^2 + {a^2}_i)$ for $2 \le i \le N$ given $a_2, \ldots, a_N$ as inputs.
Demonstrate your algorithm with $a_i = 0, Z_0 = 0$ for $N = 4$

B) Describe how to compute $z_1, \ldots, z_n$ in $O(\log N)$ steps using $O\left(\frac{N}{\log N}\right)$ PEs.

C) What is the amount of total work of the parallel machine?

D) Can you improve the algorithm?

6.0 Question 6

A) The process of checking to see if a given sequence of symbols consisting of "(" and ")" represents a balanced parenthesis is fundamental to parsing. Given

such a sequence of length n stored in a hypercube based SIMD machine, design an algorithm which determines if the string is balanced in $O(\log n)$ time.

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B) Describe an algorithm (use GPC) to find the mate for each symbol.

7.0 Question 7

Let $S = \{1, \ldots, n\}$ be the input of an ε-halver.
Prove that the number of strangers is at most $M \bullet \varepsilon$ for a subset $\{1.M\}$ where $M \leq \frac{n}{2}$.

8.0 Question 8

A) Show how to realize the permutation (3 1 4 5 6 2 0 7) in a Benes network. Demonstrate each step.

B) Can you convert the Benes network into a sorting network by replacing each switch with a comparitor?

C) Can you convert any sorting network into a permutation network by converting each comparitor into a switching element?